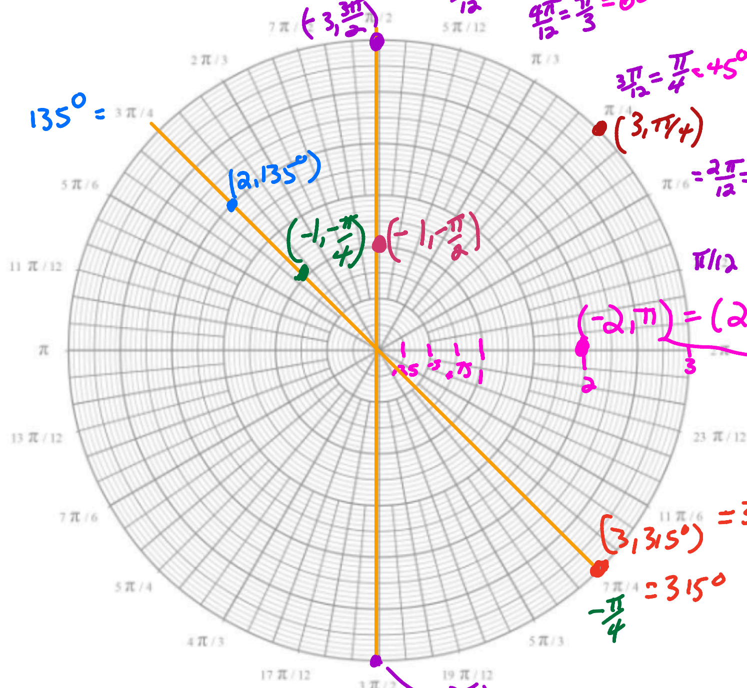
Period 4, Feb 18, 2025

a. $(2, 135°)$ **b.** $\left(-3, \frac{3\pi}{2}\right)$ **c.** $\left(-1, -\frac{\pi}{4}\right)$.

$\left(3, \frac{\pi}{2}\right)$

a. $(3, 315°)$ **b.** $(-2, \pi)$ **c.** $\left(-1, -\frac{\pi}{2}\right)$.

$\left(3, \frac{\pi}{4}\right)$.



$\frac{6\pi}{12} = \frac{\pi}{2} = 90°$ $\frac{5\pi}{12} = 75°$ $\frac{4\pi}{12} = \frac{\pi}{3} = 60°$ $\frac{3\pi}{12} = \frac{\pi}{4} = 45°$ $= \frac{2\pi}{12} = \frac{\pi}{6} = 30°$ $\pi/12$ $15°$

$135° = 3\pi/4$ $(2, 135°)$ $\left(-1, -\frac{\pi}{4}\right)$ $\left(-1, -\frac{\pi}{2}\right)$ $\left(-3, \frac{3\pi}{2}\right)$ $\left(3, \pi/4\right)$

$(-2, \pi) = (2, 0), (2, 2\pi), (-2, 3\pi)$

around the circle

around circle

$\left(3, 315°\right)$ $= 330°$ $= 315°$ $-\frac{\pi}{4}$ $-\frac{\pi}{2}$ $\left(3, \frac{3\pi}{2}\right)$

Multiple Representations of Points

If n is any integer, the point (r, θ) can be represented as

$$(r, \theta) = (r, \theta + 2n\pi) \quad \text{or} \quad (r, \theta) = (-r, \theta + \pi + 2n\pi).$$

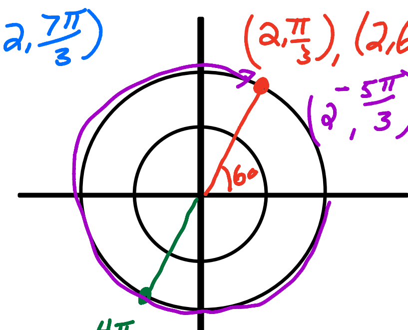
The point $\left(2, \frac{\pi}{3}\right)$ is plotted in **Figure 6.23**. Find another representation of this point in which

a. r is positive and $2\pi < \theta < 4\pi$. $\left(2, 2\pi + \frac{\pi}{3}\right)$ $\left(2, \frac{7\pi}{3}\right)$

b. r is negative and $0 < \theta < 2\pi$. $\left(-2, 4\pi/3\right)$

c. r is positive and $-2\pi < \theta < 0$. $\left(2, -\frac{5\pi}{3}\right)$

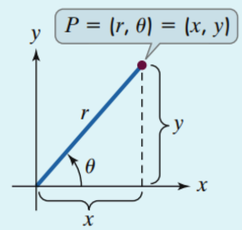
$-2\pi + \frac{\pi}{3} = \frac{-6\pi}{3} + \frac{\pi}{3} = \frac{-5\pi}{3}$



$\left(2, \frac{\pi}{3}\right), (2, 60°)$ $\left(-2, \frac{4\pi}{3}\right)$ $\left(2, -\frac{5\pi}{3}\right)$ 60 $\frac{4\pi}{3}$

Relations between Polar and Rectangular Coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$x^2 + y^2 = r^2$$
$$\tan\theta = \frac{y}{x}$$



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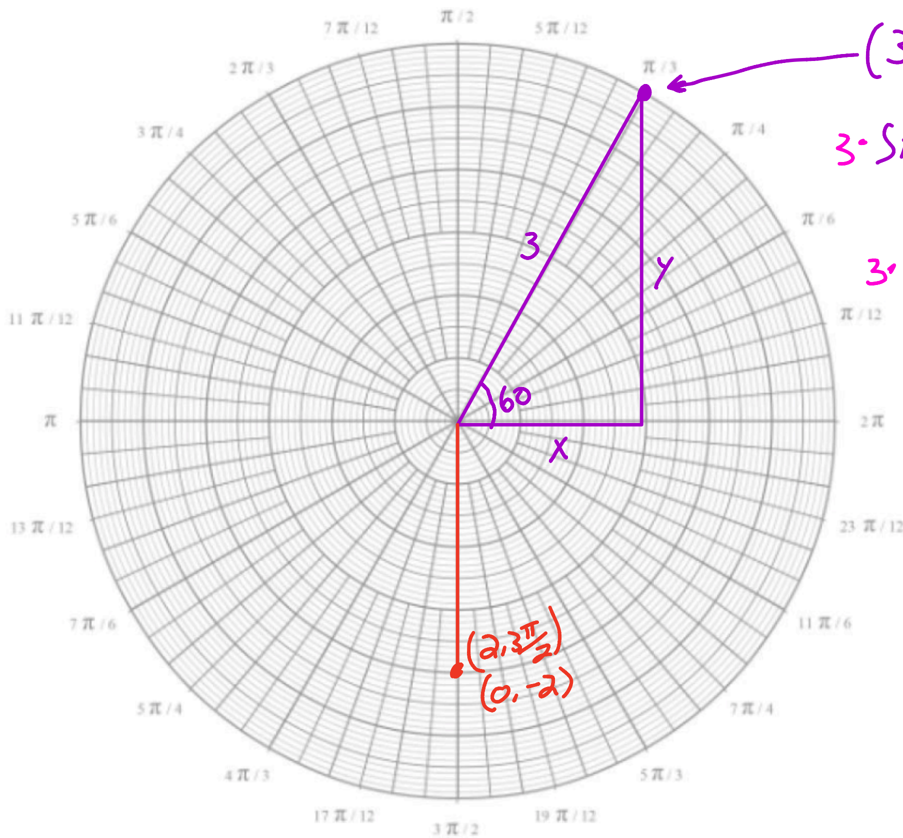


$(3, \frac{\pi}{3})$

$3 \cdot \sin 60 = \frac{y}{\cancel{3}} \cdot \cancel{3} \Rightarrow 3\sin 60 = y$

$3 \cdot \cos 60 = \frac{x}{\cancel{3}} \cdot \cancel{3} \Rightarrow 3\cos 60 = x$

$x = r\cos\theta$

$y = r\sin\theta$

Find the rectangular coordinates of the points with the following polar coordinates:

a. $\left(2, \frac{3\pi}{2}\right)$ **b.** $\left(-8, \frac{\pi}{3}\right)$.

$x = 2 \cdot \cos\frac{3\pi}{2}$

$0 = 2 \cdot 0$

$y = 2\sin\frac{3\pi}{2} = 2 \cdot -1 = -2$

$(0, -2)$

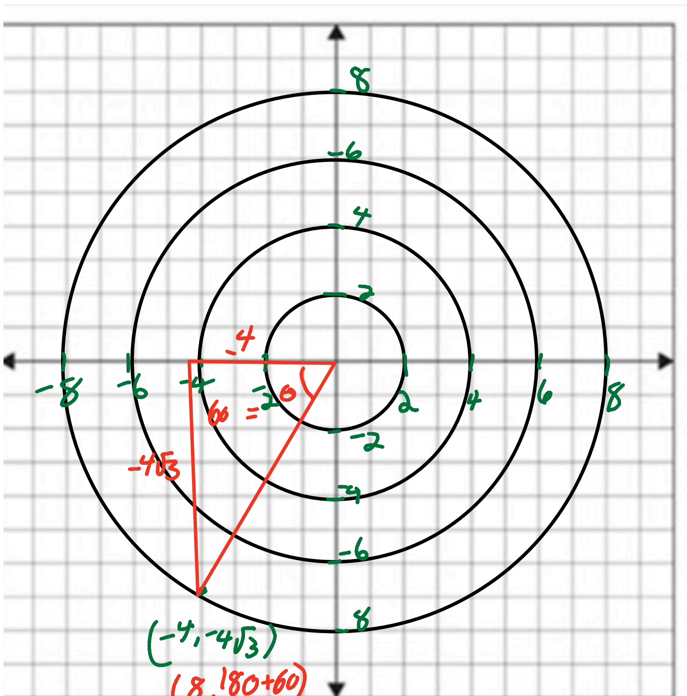
$x = -8\cos\frac{\pi}{3} = -8 \cdot \frac{1}{2} = -4$

$y = -8\sin\frac{\pi}{3} = -8 \cdot \frac{\sqrt{3}}{2} = -4\sqrt{3}$

Converting a Point from Rectangular to Polar Coordinates
($r > 0$ and $0 \le \theta < 2\pi$)

1. Plot the point (x, y).
2. Find r by computing the distance from the origin to (x, y): $r = \sqrt{x^2 + y^2}$.
3. Find θ using $\tan\theta = \frac{y}{x}$ with the terminal side of θ passing through (x, y).

Find polar coordinates of the point whose rectangular coordinates are $\left(-1, \sqrt{3}\right)$.



$(-4, -4\sqrt{3})$

$(8, 180+60)$

$(8, 240)$

$\left(-8, \frac{\pi}{3}\right), (8, 240^\circ)(8, \frac{4\pi}{3})$

$x = -8\cos\frac{\pi}{3} = -8 \cdot \frac{1}{2} = -4$

$y = -8\sin\frac{\pi}{3} = -8 \cdot \frac{\sqrt{3}}{2} = -4\sqrt{3} = 6.92$

$(-4, -6.92)$

$\tan\theta = \frac{-4\sqrt{3}}{-4} = \sqrt{3} = \tan\theta = \frac{y}{x}$

$\tan^{-1}\sqrt{3} = 60^\circ$ or $\pi/3$

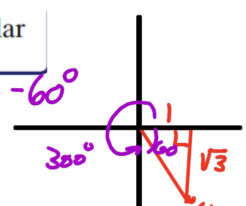
CHECK POINT 4 Find polar coordinates of the point whose rectangular coordinates are $\left(1, -\sqrt{3}\right)$.

$x = r\cos\theta$

$y = r\sin\theta$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -60^\circ$

$x^2 + y^2 = r^2 \qquad 1^2 + (\sqrt{3})^2 = r^2 \Rightarrow 1 + 3 = r^2 \Rightarrow 4 = r^2 \Rightarrow r = \pm 2$



$(-2, \frac{2\pi}{3})$ $(2, -60)$

$(-2, 120^\circ)$ $(2, 300)$

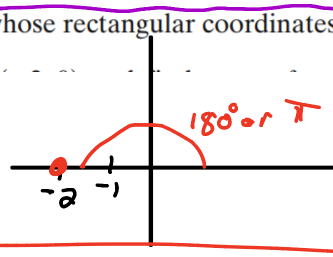
$(2, \frac{5\pi}{3})$

$(2, -\frac{\pi}{3})$

Find polar coordinates of the point whose rectangular coordinates are $(-2, 0)$.

$(2, 180)$ or $(2, \pi)$

$(-2, 0)$



A **polar equation** is an equation whose variables are r and θ. Two examples of polar equations are

$$r = \frac{5}{\cos\theta + \sin\theta} \quad \text{and} \quad r = 3\csc\theta.$$

$(\cos\theta+\sin\theta)\, r = \frac{5}{\cancel{(\cos\theta+\sin\theta)}}\,\cancel{(\cos\theta+\sin\theta)}$

$r\cos\theta + r\sin\theta = 5$

$x + y = 5$

Line $y = -x + 5$

$r = 3 \cdot \frac{1}{\sin\theta} \Rightarrow r = \frac{3}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta}$

$r\sin\theta = 3$

$y = 3$

Convert each rectangular equation to a polar equation that expresses r in terms of θ:

a. $x + y = 5$ **b.** $(x - 1)^2 + y^2 = 1$.

$r\cos\theta + r\sin\theta = 5$

$\frac{r\cancel{(\cos\theta+\sin\theta)}}{\cancel{(\cos\theta+\sin\theta)}} = \frac{5}{(\cos\theta+\sin\theta)}$

$r = \frac{5}{\cos\theta+\sin\theta}$

$(x-1)^2 + y^2 = 1$

$x^2 - 2x + \cancel{1} + y^2 = \cancel{1}$

$-1 \quad -1$

$x^2 + y^2 - 2x = 0$

$r^2 - \cancel{2r\cos\theta} = 0$

$+2r\cos\theta \quad +2r\cos\theta$

$\frac{r^2}{r} = \frac{2\cancel{r}\cos\theta}{\cancel{r}}$

$r = 2\cos\theta$

CHECK POINT 6 Convert each rectangular equation to a polar equation that expresses r in terms of θ:

a. $3x - y = 6$ **b.** $x^2 + (y + 1)^2 = 1$.

$3(r\cos\theta) - r\sin\theta = 6$

$\frac{r\cancel{(3\cos\theta - \sin\theta)}}{\cancel{3\cos\theta - \sin\theta}} = \frac{6}{3\cos\theta - \sin\theta}$

$r = \frac{6}{3\cos\theta - \sin\theta}$

$\underbrace{x^2 + y^2}_{r^2} + 2y + \cancel{1} = \cancel{1}$

$-1 \quad -1$

$r^2 + 2 \cdot \cancel{r}\sin\theta = 0$

$-2r\sin\theta \quad -2r\sin\theta$

$\frac{r^2}{r} = \frac{-2\cancel{r}\sin\theta}{\cancel{r}}$

$r = -2\sin\theta$

$\theta = \frac{\pi}{4}$

$\tan\theta = \frac{y}{x}$

$\tan\frac{\pi}{4} = \frac{y}{x}$

$x \cdot 1 = \frac{y}{\cancel{x}} \cdot \cancel{x} \Rightarrow x = y$

$r = \frac{3}{\sin\theta}$

$r\sin\theta = 3$

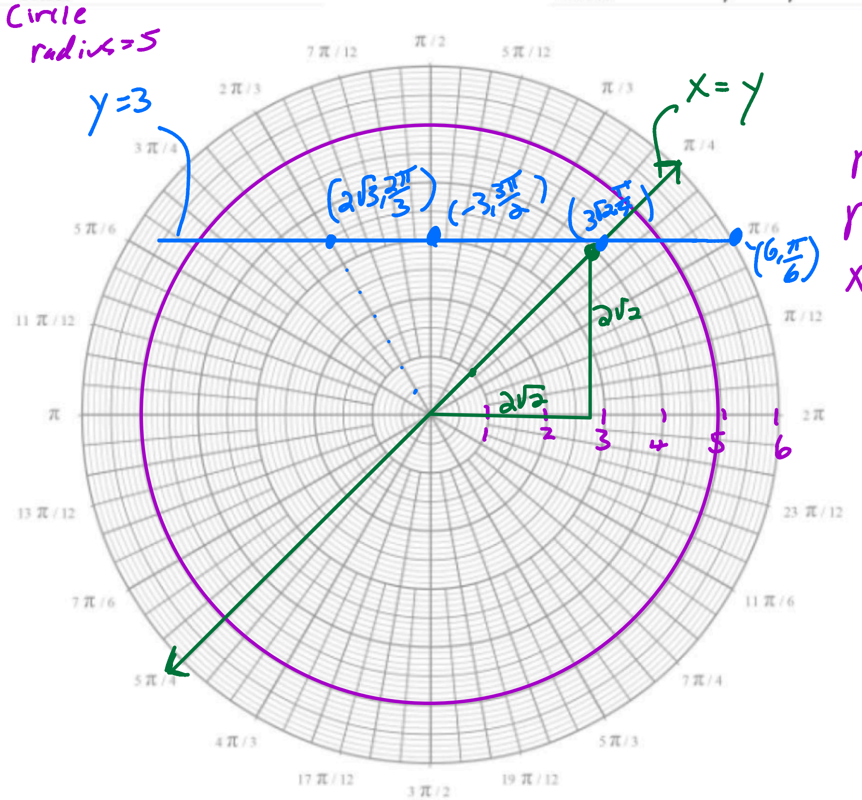
$y = 3$

$r \cdot r = -6\cos\theta \cdot r \rightarrow x^2+y^2=-6x$

$r^2 = -6r\cos\theta$

a. $r = 5$ **b.** $\theta = \dfrac{\pi}{4}$ **c.** $r = 3\csc\theta$ **d.** $r = -6\cos\theta$.

Circle radius = 5



$r=5$

$r^2=25$

$x^2+y^2=25$

circle

$\theta = \frac{\pi}{4}$

$x = r\cos\theta$

$y = r\sin\theta$

$\Downarrow$

$x = r\cos\frac{\pi}{4}$

$y = r\sin\frac{\pi}{4}$

$\Downarrow$

$x = \frac{\sqrt{2}}{2} \cdot r$

$y = \frac{\sqrt{2}}{2} \cdot r$

$\Downarrow$

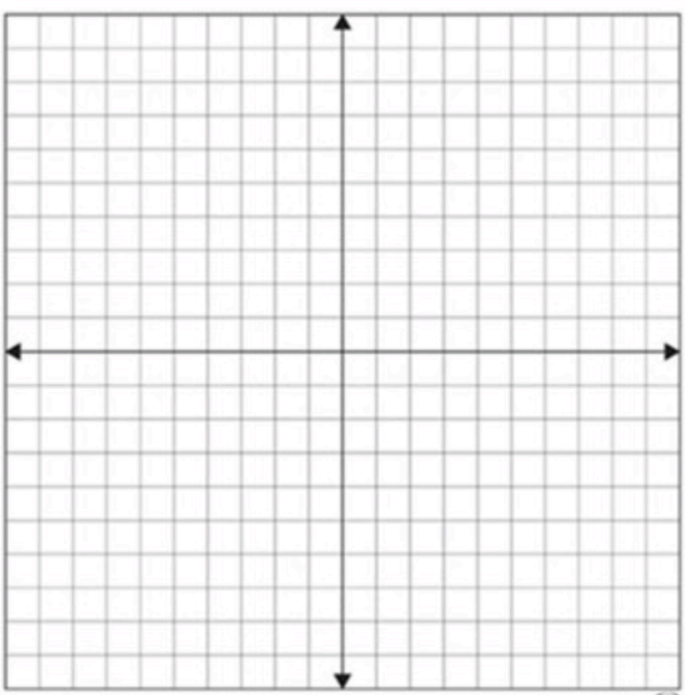
$r = 4$

$x = \frac{\sqrt{2}}{2} \cdot 4 = 2\sqrt{2}$

$y = \frac{\sqrt{2}}{2} \cdot 4 = 2\sqrt{2}$

(x, y)

$(2.828, 2.828)$



$r = \frac{3}{\sin\theta}$

θ	r
0	$\varnothing = \frac{3}{0}$
$\frac{\pi}{6}$	$6 = \frac{3}{\sin\frac{\pi}{6}} = \frac{3}{\frac{1}{2}}$
$\frac{\pi}{4}$	$3\sqrt{2} = \frac{3}{\sin\frac{\pi}{4}} = \frac{3}{\frac{\sqrt{2}}{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ 4.24
$\frac{2\pi}{3}$	$3.46 = \frac{3}{\sin\frac{2\pi}{3}} = \frac{3}{\frac{\sqrt{3}}{2}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$
$\frac{3\pi}{2}$	$-3 = \frac{3}{\sin\frac{3\pi}{2}} = \frac{3}{-1}$

$x^2+y^2=-\cancel{6x}$
$+6x \qquad \cancel{+6x}$

$x^2+6x+9+y^2=0+9 \Rightarrow \underline{x^2+6x+9}+y^2=9$

$a=1$
$b=6$
$\frac{b}{2}=\frac{6}{2}=3 \Rightarrow \left(\frac{b}{2}\right)^2=(3)^2=9$

$(x+3)^2+y^2=9$

Circle center $(-3,0)$
radius $=3$

The polar coordinates of a point are given. Find the rectangular coordinates of this point.

(r,θ)
(2.7,5.4)

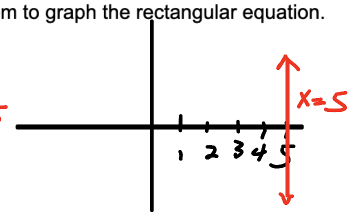
$x=2.7\cos 5.4=1.71$

$y=2.7\sin 5.4=-2.09$

Convert the polar equation to a rectangular equation. Then use a rectangular coordinate system to graph the rectangular equation.

r = 5 **sec** θ

$r=5\cdot\frac{1}{\cos\theta} \Rightarrow r=\frac{5}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta} \Rightarrow r\cos\theta=5 \Rightarrow x=5$



Convert the polar equation to a rectangular equation. Then use a rectangular coordinate system to graph the rectangular equation.

r = 12 **cos** θ + 8 **sin** θ

$r\cdot r=r(12\cos\theta+8\sin\theta)$

$r^2=12r\cos\theta+8r\sin\theta$

$x^2+y^2=12x+8y \Rightarrow x^2-12x+36+y^2-8y+16=0+36+16$
$-12x \quad -8y \quad \cancel{-12x} \quad \cancel{-8y}$

$a=1$
$b=-12$
$\frac{b}{2}=\frac{-12}{2}=-6$
$\left(\frac{b}{2}\right)^2=(-6)^2=36$

$a=1$
$b=-8$
$\frac{b}{2}=\frac{-8}{2}=-4$
$\left(\frac{b}{2}\right)^2=(-4)^2=16$

$(x-6)^2+(y-4)^2=52$

circle center $(6,4)$
radius $=\sqrt{52}=2\sqrt{13}$